Energy conservation methods for Hamiltonian Boundary Value Problems

Pierluigi Amodio, Luigi Brugnano and Felice Iavernaro

We present a few preliminary results about the implementation of energy conserving Runge-Kutta methods to the solution of Hamiltonian boundary value problems

$$\dot{y} = J\nabla H(y), \qquad g(y(0), y(T)) = 0, \tag{1}$$

where $J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$ (I is the identity matrix of dimension m), the scalar function $H : \Omega \subset \mathbb{R}^{2m} \to \mathbb{R}$ is the *Hamiltonian* of the problem and its value is constant along any solution of (1), and $g : \mathbb{R}^{2m} \times \mathbb{R}^{2m} \to \mathbb{R}^{2m}$ defines the boundary conditions.

These methods, named Hamiltonian Boundary Value Methods (HBVMs), can be interpreted as an extension of the well-known Gauss-Legendre methods with the difference that HBVMs provide a precise energy conservation for polynomial Hamiltonian functions of any high degree and, hence, a practical energy conservation for general Hamiltonian functions.

Interest in problems such as (1) arises in several research areas. We focus our attention on some applications in celestial mechanics and astrodynamics. In particular, we consider the dynamics of a massless object (planetoid) subject to the gravitational field induced by two massive bodies (primaries) revolving in circular orbits about their center of mass. Such a dynamical system, referred to as the circular restricted three-body problem, together with its generalizations, has been deeply studied since Poincaré. Its renewed interest is motivated by the fundamental role it plays in the context of space mission design and control problems in aerospace engineering, such as the nonlinear trajectory optimization and the spacecraft orbit transfer.

References

[1] P. Amodio, L. Brugnano, F. Iavernaro. Energy conservation in the numerical solution of Hamiltonian boundary value problems. AIP Conference Proceedings 1558, 35–38.

[2] P. Amodio, L. Brugnano, F. Iavernaro. Energy-conserving methods for Hamiltonian Boundary Value Problems and applications in astrodynamics. Submitted.

[3] R.H. Battin. An introduction to the mathematics and methods of astrodynamics. Revised edition. American Institute of Aeronautics and Astronautics (AIAA), Reston, VA, 1999.

[4] W.S. Koon, M.W. Lo, J.E. Marsden and S.D. Ross. Dynamical systems, the three-body problem and space mission design. Available at url: `http://www.shaneross.com/books/space`