Fractional Generalized Adams Methods for FDEs

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An initial value problem for a Fractional Differential Equation (FDE) of the Caputo type can be reformulated as a Volterra integral equation of the second kind with weakly singular kernel. Its solution can be therefore approximated by applying suitable convolution quadratures. The most studied approaches to derive them currently available in the literature are the Adams product quadrature rules and the Fractional Linear Multistep Methods, [3, 4, 5]. The resulting schemes suffer from the usual order barrier for A-stable methods. In particular, in [4] it was proved that the order of an A-stable convolution quadrature cannot exceed two. Clearly, this result represents an extension of the second Dahlquist barrier for linear multistep methods (LMMs) for ordinary differential equations. This barrier has been overcomed by using LMMs as Boundary Value Methods (BVMs), namely by coupling the difference equation provided by a LMM with suitable boundary conditions, [2]. By virtue of this result, we have investigated if the BVM approach is successful in overcoming the barrier established in [4]. This led us to derive a generalized version of Adams product quadrature rules, based on Generalized Adams Methods [2], that we have called *Fractional Generalized Adams Methods*, [1].

In this talk, we shall present these new schemes and discuss their convergence and linear stability properties. In particular, we shall show that they are always A-stable in a generalized sense. Finally, results of numerical experiments confirming the developed theory will be reported.

References

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